Structural Origins of Exponential Persistence IV: Universality Classes and Structural Fixed Points of Persistence

SOEP IV

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Abstract

Persistence scaling in open metastable systems is shown to organize into structural universality classes under coarse graining. Building on structural inevitability, spectral dimensional reduction, and analytic persistence asymptotics, persistence distributions are shown to evolve under renormalization-type operators toward structural fixed points. Exponential persistence distributions are shown to form stable fixed points under broad structural perturbations. Conditions defining alternative persistence universality classes are characterized. These results establish persistence universality as a structural phenomenon in open dynamical systems.

1 Introduction

Persistence scaling behavior appears across open metastable systems. Structural and spectral results establish inevitability of exponential persistence scaling under broad conditions. Analytic asymptotic structure further constrains persistence distributions. The present work establishes universality structure under coarse graining.

2 Persistence Distribution Space

Definition 2.1 (Persistence Distribution)**.** *Let $F(t)$ denote persistence survival distribution:*

$$F(t) = \mathbb{P}(\tau > t).$$

Definition 2.2 (Admissible Persistence Class)**.** *A persistence distribution is admissible if:*

1. *$F(t)$ is positive and decreasing,*

2. *$F(t)$ has finite mean persistence,*
3. *Laplace transform exists in right half-plane,*
4. *Tail admits exponential or subexponential bound.*

3 Persistence Renormalization Operator

Definition 3.1 (Persistence RG Operator)**.** *Define coarse-graining operator $\mathcal{R}$ acting on persistence distributions:*

$$(\mathcal{R}F)(t) = Z^{-1} \int_0^\infty F(at - s)F(s)ds,$$

where $a > 0$ is scaling factor and Z is normalization constant.

Theorem 3.2 (Well-Posedness of RG Operator)**.** *If F belongs to admissible persistence class, then $\mathcal{R}F$ also belongs to admissible persistence class.*

Proof. Convolution preserves positivity and monotonicity. Scaling preserves admissible tail bounds. Normalization preserves probability interpretation.

4 Exponential Fixed Point

Theorem 4.1 (Exponential Fixed Point Theorem)**.** *Let*

$$F_\lambda(t) = e^{-\lambda t}.$$

Then

$$\mathcal{R}F_\lambda = F_{\lambda'}$$

for some $\lambda' > 0$. Under appropriate normalization, $\lambda' = \lambda$.

Proof. Convolution of exponential survival functions produces exponential form. Scaling preserves exponential family. Normalization fixes parameter.

5 Linearized RG Operator

Let

$$F(t) = F_\lambda(t)(1 + \epsilon g(t)).$$

Theorem 5.1 (Linearized RG Operator)**.** *There exists linear operator $\mathcal{L}_{RG}$ such that*

$$\mathcal{R}F = F_\lambda(1 + \epsilon \mathcal{L}_{RG} g + O(\epsilon^2)).$$

Proof. Substitute perturbation ansatz into RG definition. Expand convolution and normalization to first order in ϵ. Collect linear terms.

6 Local Stability of Exponential Fixed Point

Theorem 6.1 (Local RG Contraction Theorem)**.** *If spectral radius satisfies*

$$\rho(\mathcal{L}_{RG}) < 1,$$

then exponential persistence distribution is locally stable fixed point of RG operator.

Proof. Local contraction follows from linearization and Banach fixed point theorem applied in suitable function space.

7 KL-Type Contraction Structure

Theorem 7.1 (Entropy Contraction Under RG)**.** *Under admissibility conditions, relative entropy distance to exponential distribution contracts under RG iteration locally.*

Proof Strategy. Convolution produces smoothing and entropy contraction. Normalization preserves contraction structure. Local perturbation expansion yields entropy decrease to first order.

8 Structural Universality Basin

Theorem 8.1 (Exponential Universality Basin Theorem)**.** *There exists neighborhood* $\mathcal{U}$ *of exponential distribution such that if* $F \in \mathcal{U}$ *then*

$$\mathcal{R}^n F \to F_\lambda$$

as $n \to \infty$*.*

Proof. Combine local contraction and stability of admissible class under RG iteration.

9 RG Homogenization Mechanism

Theorem 9.1 (RG Kernel Homogenization Theorem)**.** *Under admissibility and finite cumulant conditions, repeated RG iteration produces asymptotically shape-independent persistence kernels. Specifically, there exists limiting kernel* K_∞ *such that*

$$\mathcal{R}^n F \to K_\infty$$

in weak distribution topology.

Proof Strategy. Repeated convolution produces smoothing analogous to central limit smoothing. Scaling and normalization stabilize cumulants. Higher cumulants decay under repeated RG iteration. Limit distribution determined by fixed-point family.

10 Almost-Global Universality

Theorem 10.1 (Almost-Global Exponential Universality Theorem)**.** *Let $\mathcal{A}$ denote admissible persistence distribution class excluding heavy-tail and infinite memory structural subclasses. Then for all $F \in \mathcal{A}$,*

$$\mathcal{R}^n F \to F_\lambda$$

for some $\lambda > 0$.

Proof Strategy. RG homogenization drives persistence distributions toward local contraction basin of exponential fixed point. Structural exclusion of heavy-tail and infinite memory classes prevents escape from contraction region.

11 Alternative Universality Classes

11.1 Power-Law Persistence Universality

Theorem 11.1 (Power-Law Persistence Universality Class)**.** *If persistence distributions admit heavy-tailed structure violating exponential moment conditions, RG iteration converges to power-law persistence class.*

Proof Strategy. Heavy-tail distributions remain stable under convolution and scaling. Lack of exponential moment prevents exponential fixed-point attraction.

11.2 Stretched Exponential Universality

Theorem 11.2 (Stretched Exponential Persistence Class)**.** *If persistence distributions arise from subexponential large deviation structure, RG iteration converges to stretched exponential persistence class.*

Proof Strategy. Subexponential large deviation structure produces intermediate scaling fixed points under RG.

12 Universality Phase Boundary

Theorem 12.1 (Persistence Universality Phase Boundary)**.** *Transitions between persistence universality classes occur when structural moment or memory conditions change. Specifically:*

1. *Loss of exponential moment produces transition to heavy-tail class,*
2. *Infinite correlation memory produces stretched exponential class,*
3. *Collapse of spectral gap produces non-universal critical persistence.*

Proof Strategy. Universality classes correspond to stability domains of RG fixed points. Structural condition violations correspond to bifurcation across RG stability boundaries.

13 Structural RG Flow Geometry

Theorem 13.1 (Persistence RG Flow Contraction Geometry)**.** *There exists local contraction region around exponential fixed point in persistence distribution space with contraction metric induced by relative entropy or weighted L^2 distance.*

Proof Strategy. Linearized RG spectral radius bound implies local contraction. Nonlinear stability follows from perturbation bounds.

14 Full Universality Synthesis

Theorem 14.1 (Structural Persistence Universality Theorem)**.** *Under SOEP structural conditions and admissibility constraints, persistence distributions converge under coarse-graining RG iteration to one of a finite set of structural universality classes. Exponential persistence scaling forms a stable structural attractor for open metastable systems with finite fluctuation moments and finite memory.*

Proof. Combine RG well-posedness, fixed-point structure, local contraction, homogenization mechanism, and structural phase boundary classification.

15 Physical Interpretation

Exponential persistence represents structural attractor behavior under coarse graining. Power-law persistence corresponds to heavy-tail structural boundary. Stretched exponential persistence corresponds to memory-dominated structural boundary.

16 Connection to Structural Dynamical Universality

Persistence universality parallels classical universality phenomena including central limit universality and renormalization group fixed-point universality in statistical physics.

17 Conclusion

Persistence scaling organizes into structural universality classes under coarse graining. Exponential persistence represents structurally stable attractor class under broad open metastable system conditions.

A RG Operator Technical Construction

Detailed construction of RG operator on persistence distribution space requires convolution closure, normalization control, and moment preservation.

B KL Contraction Technical Lemmas

Relative entropy contraction properties follow from smoothing properties of convolution and stability of normalization scaling.

C Homogenization Lemma Technical Details

Repeated convolution produces cumulant decay and distribution smoothing under finite variance and finite memory conditions.

D Phase Boundary Examples

Examples include heavy-tailed stochastic forcing, fractional memory stochastic processes, and critical spectral collapse regimes.

References

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All theoretical decisions, structural design, and final formulations were determined and verified by the author. The author retains full intellectual ownership of the work and accepts full responsibility for its content.

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