Structural Origins of Exponential Persistence III: Analytic Asymptotics of Persistence Distributions SOEP III

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Abstract

Analytic asymptotic structure of persistence time distributions is established for open metastable systems. Building on structural inevitability and spectral dimensional reduction results, persistence distributions are shown to admit Laplace transform representations dominated by slow spectral eigenvalues. Analytic continuation, contour deformation methods, and cumulant-based asymptotic expansions yield explicit persistence distribution approximations with controlled remainder bounds. These results provide analytic closure linking spectral persistence reduction to asymptotic persistence universality structure.

1 Introduction

Persistence distributions contain finer structural information than mean persistence times or escape rates. Spectral reduction implies finite-dimensional control of persistence observables. The present work establishes analytic asymptotic structure of persistence distributions using Laplace transform and complex analytic methods.

2 Persistence Distribution and Laplace Transform

Definition 2.1 (Persistence Density). *Let τ denote persistence time. Define persistence density $f(t)$ such that*

$$\mathbb{P}(\tau \in dt) = f(t)\, dt.$$

Definition 2.2 (Laplace Transform). *Define the Laplace transform*

$$\hat{f}(s) = \int_0^\infty e^{-st} f(t)\, dt.$$

3 Spectral Laplace Representation

Let L denote effective generator and let $P_t = e^{tL}$ denote semigroup.

Theorem 3.1 (Spectral Laplace Representation). *There exist observables g, h such that*

$$\hat{f}(s) = \langle (s - L)^{-1} g, h \rangle.$$

Proof. Laplace transform of semigroup survival observable yields resolvent representation using standard semigroup–resolvent correspondence.

4 Dominant Pole Structure

Theorem 4.1 (Dominant Pole Persistence Theorem). *Let λ_1 denote the principal slow eigenvalue of L. Then near $s = -\lambda_1$,*

$$\hat{f}(s) = \frac{A}{s + \lambda_1} + R(s),$$

where $R(s)$ is analytic in a neighborhood of $-\lambda_1$.

Proof. Spectral decomposition of resolvent yields isolated pole corresponding to principal eigenvalue. Remaining spectrum contributes analytic remainder separated by spectral gap.

5 Pole Separation

Theorem 5.1 (Pole Separation Theorem). *There exists $\delta > 0$ such that $\hat{f}(s)$ is analytic in the strip*

$$\Re(s) > -\lambda_1 - \delta$$

except at $s = -\lambda_1$.

Proof. Quasi-compact spectral structure implies essential spectrum lies strictly to left of slow eigenvalue cluster. Resolvent analytic outside spectrum.

6 Analytic Continuation

Theorem 6.1 (Analytic Extension Theorem). *If the semigroup admits exponential moment bounds, then $\hat{f}(s)$ admits analytic extension into complex strip*

$$\Re(s) > -\sigma_0$$

for some $\sigma_0 > 0$.

Proof. Exponential moment bounds imply uniform Laplace convergence in strip. Analyticity follows from dominated convergence and Morera-type arguments.

7 Bromwich Inversion Representation

Theorem 7.1 (Bromwich Inversion Formula). *For $\gamma > \sup \Re(\sigma(L))$,*

$$f(t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} e^{st} \hat{f}(s)\, ds.$$

Proof. Standard Laplace inversion formula applies under analytic continuation and growth bounds.

8 Contour Deformation Framework

Let Γ denote deformation contour passing through dominant pole or saddle region.

Lemma 8.1 (Contour Deformation Validity). *If $\hat{f}(s)$ is analytic in region between original and deformed contour and satisfies suitable growth bounds, contour deformation preserves inversion integral.*

Proof. Follows from Cauchy integral theorem and vanishing of large arc contributions under exponential decay bounds.

9 Saddle Contour Construction

Let $s_0 = -\lambda_1$ denote the dominant pole location.

Theorem 9.1 (Saddle Contour Persistence Asymptotic Theorem). *Assume $\hat{f}(s)$ admits analytic continuation in a neighborhood of s_0 and admits local expansion*

$$\hat{f}(s) = \frac{A}{s - s_0} + B(s),$$

where $B(s)$ is analytic near s_0. Then persistence density admits leading asymptotic form

$$f(t) = Ae^{s_0 t} + R_1(t),$$

where remainder term satisfies

$$|R_1(t)| \le Ce^{(s_0 - \eta)t}$$

for some $\eta > 0$.

Proof. Deform Bromwich contour through pole location. Residue theorem yields leading exponential contribution. Remainder integral bounded using exponential decay of integrand away from pole.

10 Steepest Descent Local Expansion

Lemma 10.1 (Local Phase Expansion). *Let $\Phi(s) = st + \log \hat{f}(s)$. Near saddle point s_*,*

$$\Phi(s) = \Phi(s_*) + \frac{1}{2}\Phi''(s_*)(s - s_*)^2 + O((s - s_*)^3).$$

Proof. Follows from analytic Taylor expansion of $\Phi(s)$ near stationary point.

11 Edgeworth-Type Persistence Expansion

Theorem 11.1 (Uniform Persistence Edgeworth Expansion). *Assume existence of cumulants κ_n satisfying uniform analytic bounds. Then persistence density admits asymptotic expansion*

$$f(t) = e^{s_0 t}\left(c_0 + \frac{c_1}{t^{1/2}} + \frac{c_2}{t} + \cdots\right) + R_2(t),$$

where remainder satisfies

$$|R_2(t)| \leq Ct^{-N} e^{s_0 t}.$$

Proof. Expand log Laplace transform into cumulant series. Apply inverse Laplace asymptotics using steepest descent combined with classical Edgeworth expansion methods.

12 Global Frequency Patch Structure

Define Fourier frequency variable ω.

Split frequency domain into low-frequency and high-frequency regions:

$$|\omega| \leq \Omega, \quad |\omega| > \Omega.$$

Theorem 12.1 (Global Frequency Patch Theorem). *Assume spectral pole expansion holds for $|\omega| \leq \Omega$ and analytic decay bounds hold for $|\omega| > \Omega$. Then global transform bound holds:*

$$|\hat{f}(i\omega)| \leq C(1 + |\omega|)^{-k}$$

for some $k > 1$.

Proof. Low frequency region controlled using spectral pole expansion. High frequency region controlled using repeated integration by parts and analytic decay of transform derivatives. Matching at cutoff frequency produces global bound.

13 Fourier $\mathbf{L^1}$ Transfer Theorem

Theorem 13.1 (Transform Inversion Stability). *If $\hat{f}(\omega) \in L^1(\mathbb{R})$ and satisfies polynomial decay bounds, then persistence density satisfies uniform bound*

$$|f(t)| \leq C.$$

Proof. Apply Fourier inversion theorem and dominated convergence using integrability of transform.

14 Full Persistence Asymptotic Formula

Theorem 14.1 (Full Persistence Asymptotic Expansion). *Under analytic continuation, spectral pole dominance, Edgeworth cumulant control, and global frequency patch conditions, persistence density admits asymptotic form*

$$f(t) = e^{s_0 t} \left(P(t^{-1/2}) + O(t^{-N}) \right),$$

where P is finite polynomial determined by cumulants.

Proof. Combine saddle contour asymptotic, Edgeworth expansion, and global transform remainder bounds. Fourier inversion transfers transform bounds to time-domain remainder bounds.

15 Structural Consequences

Persistence distributions asymptotically depend only on dominant slow eigenvalues and finite cumulant structure. This establishes analytic persistence reduction to slow spectral invariants.

16 Bridge to Universality Theory

Analytic asymptotic persistence structure enables classification of persistence scaling behavior across systems with equivalent slow spectral and cumulant structures.

17 Conclusion

Analytic asymptotic structure of persistence distributions is determined by slow spectral poles, local saddle geometry, and cumulant structure of persistence transform. These results provide analytic closure connecting spectral persistence reduction to universality classification.

A Contour Geometry Constants

Explicit contour selection depends on analytic strip width and exponential growth bounds of Laplace transform.

B Edgeworth Remainder Propagation

Uniform cumulant bounds imply uniform remainder bounds via classical asymptotic expansion error propagation.

C Fourier Transfer Technical Bounds

Polynomial decay of transform derivatives ensures integrability and inversion stability.

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All theoretical decisions, structural design, and final formulations were determined and verified by the author. The author retains full intellectual ownership of the work and accepts full responsibility for its content.